Topics in Advanced Analysis I
2024-2025
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Lingua insegnamento
Inglese

Obiettivi formativi
In 1908, Poincaré asked "Why is it that showers and even storms seem to come by chance, so that many people think it quite natural to pray for rain or fine weather, though they would consider it ridiculous to ask for an eclipse by prayer.". A hundred years later it can be said that we understand, to some extent, the answer to this question and this course will introduce some of the ideas and techniques involved in addressing it.

Any process that evolves in time according to some physical law is a "deterministic" dynamical system in the sense that its evolution is completely determined by the initial condition and the physical laws. However in many cases, ranging from complicated systems such as the weather to "simple" systems such as tossing a coin, they can be very unpredictable, so much so that they are sometimes referred to as "chaotic". Even more surprisingly, despite their unpredictability many systems are very predictable in a "statistical" sense, so much so that we often refer to them as "random" even though they are far from random.

The purpose of this course is to study relatively simple, but already quite complex, mathematical models of such chaotic systems. We will study them from both a "topological" point of view and an "ergodic/probabilistic" point of view, and explore the similarities and differences of describing systems with these two different set of "lenses".

By the end of the course the student should have the background to read some quite advanced texts in Dynamical Systems and to understand the formulation of some open research problems.

Prerequisiti

One of the beauties of the theory of Dynamical Systems is that it is possible to ask some deep questions, and in some cases even answer them, with non-trivial arguments which can be quite direct and do not necessarily require very advanced machinery from other areas of mathematics. The main prerequisite for this course are i) basic knowledge about metric spaces (such as convergence, limits, compactness), and ii) basic knowledge of measure theory (such as the definition of a measure and the weak-star topology, both of which will however be reviewed in the course notes).

Contenuti

The course is divided into two parts.

Part I: Fundamental Structures
A dynamical system, in its purest form, requires only a set and a map. Considering iterates of this map already gives rise to a lot of interesting and non-trivial structures. Things get even more interesting when we add a topological or a differentiable structure to the space as we are able to describe more details. Finally, dynamical systems have an algebraic group structure which formally allows us to study autonomous ordinary differential equations with very similar techniques. This part consists of 4 Chapters:
1) Set-Theoretic Structures
2) Topological Structures
3) Differentiable Structures
4) Algebraic Structures and Differential Equations

Part II: Differentiable Ergodic Theory
A fifth fundamental structure we can introduce is that of a measurable structure. This opens the doors to a completely new approach to dynamical

systems focussed on their probabilistic and statistical properties. This part consists of 4 Chapters:
5) Probabilistic Structures
6) Invariant Measures
7) Ergodic Measures
8) Full Branch Maps

Metodi didattici

The teaching methods will be quite different from usual courses. There will be very few standard blackboard lectures and most of the learning will be done instead individually or in groups, working on exercises, with the lecturer and tutor available to discuss. Students will also give presentations.

Modalità di verifica dell'apprendimento

The evaluation of the course will be based on the student presentations as well as a standard final written exam.

Altre informazioni

This is not a physics course nor a numerical/computational course. The approach will be rigorous and essentially analytical. By the end of the course the students should have sufficient mathematical background to understand the formulation of some open problems in the field. The teaching method will be quite different from standard “lecture-based” teaching and much more oriented towards learner-centred “flipped classroom” approach. The lecturer will give a limited amount of lectures but most of the material will be developed through exercises, in-call discussions, and student presentations.